

Notes 3.6 – Synthetic Division and Finding the Conjugate

Warmup – Solving for x.

1. $5x + 13 = 48$

$5x = 35$

$x = 7$

2. $\frac{1}{3}x - 8 = 0$

$\frac{1}{3}x = 8$

$x = 24$

3. $-4 - 9x = 0$

$-9x = 4$

$x = -\frac{4}{9}$

4. $x^2 - 16 = 0$

$(x+4)(x-4) = 0$

$x = 4, -4$

5. $x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3, -1$

6. $x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 2, 3$

7. $(x-5)(x-7) = 0$

$x = 5, 7$

8. $(3x-18)(5x-10) = 0$

$3x - 18 = 0$

$5x - 10 = 0$

$3x = 18$

$5x = 10$

$x = 6$

$x = 2$

Investigation – Synthetic Division

Divide $(x^4 - 2x^3 + 3x + 12) \div (x + 3)$

Long Division

$$\begin{array}{r}
 x^3 - 5x^2 + 15x - 42 \\
 x+3 \overline{) x^4 - 2x^3 + 0x^2 + 3x + 12} \\
 \underline{x^4 + 3x^3} \\
 -5x^3 + 0x^2 \\
 \underline{-5x^3 - 15x^2} \\
 15x^2 + 3x \\
 \underline{15x^2 + 45x} \\
 -42x + 12 \\
 \underline{-42x - 126} \\
 138
 \end{array}$$

$$x^3 - 5x^2 + 15x - 42 + \frac{138}{x+3}$$

Practice, write answers in Standard Form.

a. $(x^3 - 2x^2 + 3) \div (x - 2)$

$$\begin{array}{r}
 2 \overline{) 1 \ -2 \ 0 \ 3} \\
 \underline{ 2 \ 0 \ 0} \\
 1 \ 0 \ 0 \ 3
 \end{array}$$

$$x^2 + \frac{3}{x-2}$$

b. $(x^4 + 2x^3 - 3x^2 + x - 5) \div (x + 3)$

$$\begin{array}{r}
 -3 \overline{) 1 \ 2 \ -3 \ 1 \ -5} \\
 \underline{ -3 \ 3 \ 0 \ -3} \\
 1 \ -1 \ 0 \ 1 \ -8
 \end{array}$$

$$x^3 - x^2 + 1 + \frac{-8}{x+3}$$

$$x + 3 = 0$$

$$x = -3$$

coefficients of dividend

Synthetic Division

$$\begin{array}{r|rrrrr}
 -3 & 1 & -2 & 0 & 3 & 12 \\
 & & + & + & + & + \\
 & \downarrow & -3 & 15 & -45 & 126 \\
 \hline
 & 1 & -5 & 15 & -42 & 138 \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \\
 & -3 \cdot 1 = -3 & -3 \cdot -5 = 15 & & & \\
 & -3 \cdot -5 & & & &
 \end{array}$$

Zero coefficients add products of zero and coeff.
 1st 2nd 3rd etc coefficients of answer

$$x^3 - 5x^2 + 15x - 42 + \frac{138}{x+3}$$

c. $(x^3 - 2x^2 - 13x - 10) \div (x - 5)$

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -13 & -10 \\ & & 5 & 15 & 10 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$x^2 + 3x + 2$$

d. $(x^4 - 16) \div (x - 2)$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & 0 & -16 \\ & & 2 & 4 & 8 & 16 \\ \hline & 1 & 2 & 4 & 8 & 0 \end{array}$$

$$x^3 + 2x^2 + 4x + 8$$

Finding the Conjugate

A **conjugate pair** is simply a pair of binomials that have the same numbers but differ by having opposite signs between them. For example $(a + b)$ and $(a - b)$ are conjugate pairs. You've probably noticed them when you've factored a quadratic expression that is the difference of two squares.

Example: $x^2 - 25 = (x + 5)(x - 5)$. The two factors $(x + 5)(x - 5)$ are conjugate pairs.

Example: The conjugate of $(3 + \sqrt{2})$ is $(3 - \sqrt{2})$. The conjugate of $(-2i)$ is $(+2i)$. Think of it as $(0 - 2i)$ and $(0 + 2i)$. Change only the sign between the two numbers.

Find the conjugate for each value:

e. $(8 + \sqrt{5})$

$$(8 - \sqrt{5})$$

f. $(11 + 4i)$

$$(11 - 4i)$$

g. $9i$

$$(-9i)$$

h. $-5\sqrt{7}$

$$(5\sqrt{7})$$

Vocabulary

Word	Meaning/Notation	Example
Synthetic Division	a shortcut to dividing polynomials when dividing by a linear binomial	$(x^2 - 9) \div (x - 3)$
Conjugate	an identical binomial that uses the opposite operation $+ \Leftrightarrow -$	binomial: $3 - 4i$ conjugate: $3 + 4i$